Does Private Equity Systematically Over-Lever Companies? *

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Abstract

Private equity (PE) funds are often criticised for over-leveraging their portfolio companies, raising chances of bankruptcy and systemic economic risk. Using a large sample of PE deals in a matched difference-in-differences framework, I find PE-owned companies generate higher cash flows and receive additional equity injections if in distress. Motivated by these patterns, I develop a structural model of optimal capital structure in an environment where the value of PE-owned firms grows faster relative to non-PE owned companies. PE's "deep-pockets" justifies an optimally chosen bankruptcy point, which offsets agency costs associated with the fund manager's option-like payoff. The model predicts optimal leverage ratio (Debt/Asset) of portfolio companies is 55-60 percent, broadly consistent with the data. Consequently, the cost of remaining at sub-optimally lower levels of leverage are substantial, particularly when firm risk or bankruptcy costs are low. Following PE-ownership, mean probability of default rises by 3.0 percent and even less when estimated in an ultra-low interest rate environment. However, default probability rises by nearly 5.0 percent if estimated with traditional credit risk models that do not internalize unique PE-characteristics.

Keywords: Optimal Capital Structure; Private Equity; Default Risk

JEL Codes: G23, G32, G32.

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1 Introduction

Since the global financial crisis, Private Equity (PE) groups have raised more than 2.5 trillion in equity, with each dollar typically leveraged with more than two dollars of debt. Such high levels of debt in PE-sponsored leveraged buyouts (LBO) have spawned two competing views[1]. One view is that PE over-leverages firms, leading to debt-overhang (Myers, 1977) and systemic economic risk while PE funds profit handsomely using the "2 and 20" fee model [2]. This view has led policymakers such as Senator Elizabeth Warren to propose new regulations to end what she decries as "legalized looting" by investment firms that take over troubled companies [3]. The competing view refutes this claim. Drawing on the canonical trade-off theory of capital structure, it argues that observed higher leverage is optimal under PE-ownership because the expected cost of financial distress is lower (e.g. Jensen (1989); Axelson et al. (2008); Brown (2021)).

This paper comprehensively investigates if PE funds systematically over-lever the companies they acquire. There are two broad objectives: (i) estimate a structural model of optimal capital structure of PE-owned companies and compare model-implied leverage ratios with data from a large and representative sample of levered buyouts, and (ii) extract default probabilities and compare them with a benchmark credit risk model from Merton (1974).

Two underlying tensions prevent direct estimation of an "off-the-shelf" capital structure model. First, the agent making capital structure and any subsequent default-related decision is the risk-neutral PE fund manager, not the acquired company. Ignoring this distinction will lead to different estimates of optimal leverage since the PE fund manager's

[1]In an LBO, a financial acquirer takes over a company using a significant amount of debt, restructures the target, and sells it once exit opportunities become sufficiently appealing (Kaplan and Stromberg, 2009).

[2]The "two" refers to an annual management fee of two percent of the capital that investors have committed to the fund. The "twenty" refers to a twenty percent share of the future profits of the fund; this profits interest is also known as the "carry" or "carried interest." The profits interest is what gives fund managers upside potential: If the fund does well, the managers share in the treasure. If the fund does badly, however, the manager can walk away (Fleischer (2008)).

[3]https://www.cbsnews.com/news/elizabeth-warren-private-equity-firms-are-like-vampires-proposes-curbs-on-wall-street-in-new-bill/.

payoffs are different compared to a benchmark equity-holder. Second, the PE fund is not liable for LBO debt. I reconcile these issues by embedding a PE fund manager or the General Partner's (GP) option-like compensation into a model of optimal leverage.

I begin by establishing two stylized facts with a multi-country dataset of PE-backed firms, for which I observe several years of accounting information pre and post-LBO. Using a propensity-score matched control group in a difference-in-differences setting, I show that PE-backed firms tend to generate higher cash flows, invest more in intangible assets such as research and development and are more efficient. Collectively, these findings suggest PE-backed firms generate an excess return that cannot be replicated by other investors. I find that risky investments are made possible by extending the maturity structure of outstanding debt which minimizes refinancing risk. Second, I document that PE-owned firms receive additional equity injection if their earnings fall below required interest payments, i.e. if they are in distress. This finding supports the view that PE-funds have "deep-pockets." Since funds are raised from institutional investors in the form of capital commitments that are drawn down and invested over a series of years, GPs can make equity injections in their portfolio companies when accessing other sources of capital are difficult. Much of the capital structure literature endogenizes bankruptcy by assuming companies can readily issue equity, a questionable assumption for highly-levered privately held firms. However, PE's unique source of equity capital justifies an optimal default-triggering threshold or endogenous bankruptcy.

To explain these patterns, the paper shifts to a quantitative analysis of the role of PE-ownership on optimal leverage and default probability. To this end, I propose a variant of models in Leland (1994), Goldstein et al. (2001) and Sorenson et al. (2014). Under the assumption of time-independence and a stationary debt structure, I solve a GP's optimization problem subject to an endogenous bankruptcy condition. The GP's payoffs consist of an asset management fee and an incentive-based profit-sharing fee. The latter ensures the GP has an incentive to protect the portfolio company's going concern by choosing the value-maximizing level of leverage and default threshold. However, the requirement to return the equity received from external investors (Limited Partners) with a minimum required rate of return changes the default-triggering threshold from traditional models. To underscore the difference in optimal leverage due to PE-ownership, I also estimate the model for a non-PE owned company. The latter does not generate any excess return, and defaults when it breaches an exogenously-set threshold. The two types of companies are also different in terms of their risk-levels or asset volatility.

In the baseline version of the model, I find that an optimally levered PE-backed firm has a leverage ratio ($Debt/Asset$) of 60 percent, moderately higher than actual data: 53 percent median Net Debt/Asset post-buyout. Agency costs plausibly associated with the GP's option-like contract is more than offset by PE's "deep-pockets", justifying a higher optimized leverage ratio. A key factor that explains higher optimal leverage is endogenous bankruptcy: when I estimate a PE-company's optimal leverage with excess return but exogenous bankruptcy, optimal leverage ratio drops below 40 percent. As mentioned earlier, PE-backed firms can justifiably endogenize bankruptcy given its access to PE's "deep-pockets"[4]. Moreover, calibrating the model to an economy without any excess return leads to an optimal debt ratio of around 50 percent, only marginally below the median LBO leverage in the data. However, in a simulated extension of the model that introduces substantial heterogeneity in firm risk (as opposed to a single estimated value for all firms) optimal leverage declines to around 45 percent for a large cross-section of firms.

Next, I estimate the cost of remaining at leverage ratios documented in non-PE or benchmark companies. I find substantial loss in firm value if PE remains at sub-optimally lower levels of leverage. The effects are particularly pronounced if firm risk or bankruptcy cost is low with 10.6 and 6.0 percent of firm value lost respectively. Finally, I compute

[4]Bernstein et al. (2017) find results consistent with this view. Furthermore, widespread media reports suggest PE funds provided financial assistance to their portfolio companies since the onset of the COVID-19 pandemic. For example, Leonard Green & Partners created an assistance funds for employees of portfolio companies.

Distance-To-Default and expected probability of default similar to Vassalou and Xing (2004) and Bharath and Shumway (2008) using an iterative algorithm equivalent to Duan (1994, 2000). First I estimate default probability using the canonical credit risk model in Merton (1974). Using firm-specific data, I estimate the model pre and post-LBO. I find that mean default rate rises from 5.7 percent to 10.6 post-acquisition. This serves as the benchmark to understand predictions of my model. I then estimate likelihood of bankruptcy using my model featuring a GP's payoffs, endogenous bankruptycy and excess return. I find mean default probability post-LBO is much lower at 7.9 percent. These findings suggest classic credit risk models may not be adequate to explain default risk in PE-backed firms which have access to unique sources of equity capital and an alpha-generation ability that cannot be replicated by other investors. Thus traditional models may overestimate default-probabilities of PE-backed companies.

Related Literature: This paper builds on several strands of literature. First, my paper contributes to both the empirical and theoretical literature on private equity-sponsored levered buyouts. To the best of my knowledge, this is the first paper that estimates model-implied leverage in PE-backed companies and compare optimal leverage with data from a large and representative sample of buyouts. To date, papers investigating capital structure in PE are mostly empirical: Axelson et al. (2013) assess determinants of capital structure using a rich multi-country sample of LBOs. However, the authors are unable to control for pre-deal trends as well as time-varying standard firm-level debt determinants typically seen in papers on capital structure. I rely on a sample which (i) includes pre-deal information and (ii) provides a rich set of time-varying firm-level controls that can better capture changes in debt. More importantly, my structural model embeds characteristics unique to PE.

Second, my paper contritbutes to the capital structure literature, particularly Leland (1994), Leland and Toft (1996) and Goldstein et al. (2001). Traditional capital structure models cannot be directly applied to portfolio companies in PE. To the best of my knowledge, this is the first paper that offers an analytical framework that embeds PE's unique institutional structure, compensation scheme and alpha-generation ability in a traditional capital structure model to investigate optimal leverage.

Third, my paper also contributes to the literature on default risk. Bharath and Shumway (2008) estimate default probability using Merton (1974). I show that without accounting for PE's unique characteristics, traditional credit risk models such as Merton (1974) may over-estimate default probabilities.

2 Stylized Facts on PE-owned Companies

I begin the analysis by collecting a set of stylized empirical facts on PE-backed firms. Using a carefully constructed repsentative dataset of PE-firms and an identical control group, I establish the following characteristics: (i) PE-ownership raises leverage substantially; (ii) PE-ownership generates an excess return; (iii) PE-backed firms have lower risk; and (iv) PE funds inject additional equity into portfolio companies if they fall into distress.

2.1 Data and Matched Control Group

2.1.1 Data Construction

The data collection process is divided into three parts. First, similar to Jenkinson and Sousa (2015) and Jenkinson and Stucke (2011) I collect private equity deal-level data from Bureau Van Dijk's (BvD) Zephyr Merger and Acquisitions database. Zephyr has been increasingly utilized among PE researchers (Bansraj et al. (2019); Hammer et al 2017; Tykvova and Borell, 2012) and has been verified as a comprehensive and representative sample of PE transactions compared with other PE databases such as Standard and Poor's Capital IQ (Jenkinson and Stucke, 2011). I retrieve all Private Equity transactions labelled

"Institutional Buyout" [5]. Next, I add all acquisitions with transaction financing described as "private equity" and "leveraged buyout" which were undertaken by a financial sponsor or by an acquirer whose business description includes the term "private equity". I select all deals from 2010 to 2019.

Second, I match target firms with their company-level accounting data from Orbis, using BvD identifiers. I require that data is available for each variable to be used in the baseline specification for the entire 2010-2019 period. I follow Kalemli-Ozcan et al (2015) to download and clean financial data for targets in order to reduce the survivorship bias present in online Orbis downloads[6]. Using this filtering process and removing observations without the required data for the 2010-2019 period yields a total of around 3,500 firm-year PE observations for 814 verified and unique LBOs [7].

Third, I construct a control group by retrieving company-level data of all non-PE firms from Orbis for the same sample period. I require the relevant financial of control companies to be available in Orbis in at least three pre-deal year, where the deal year refers to the year a target was acquired by a private equity firm. However, my identification strategy faces challenges that are typical in empirical corporate finance due to the endogeneity of acquisition decisions. To alleviate selection issue, I match individual companies that are acquired by PE firms with non-acquired companies in the same country, sector and year to control for the common trends in the fundamentals.

2.1.2 Matching Procedure

Using a logit model, I generate the conditional treatment probability (or propensity) of receiving an LBO investment based on observable firm characteristics. To ensure that my

[5]Zephyr defines this as "an acquisition where a Private Equity firm has taken a 50 percent stake or more in the Target company, or is the parent of the Acquiror. The acquisition often takes place through a 'new company' (newco) or an acquisition vehicle."

[6]One limitation I face is that Orbis does not provide data prior to 2010 in their online interface. Due to the need to purchase earlier vintage, I follow instructions from Kalemli-Ozcan et al (2015) for the available sample of 2010-2019 only. Nevertheless, considering the large number of firm-years, the dataset has among the widest multi-country company-level coverage over a 10-year period utilized in recent studies.

[7]LBO verification is discussed in the Appendix

results are not sensitive to alternate matching criterion or loss of data due to strict matches, I carry out two propensity-score matching exercises.

First, I match each treated company to the control firm with the closest propensity score (i.e. the nearest neighbor) within each country-sector-year combination. With the diff-in-diff matching estimation, Roberts and Whited (2013) recommend to match on firm characteristics and growth rates of outcome variables to ensure similarity of pre-treatment trends.

The nature of traditional venture capital and leveraged buyout targets guides my choice of matching variables. Following Bansraj et al. (2019), for my baseline specification, I match on log of total assets, growth of sales. Since PE funds are likely to select companies based on efficiency and debt-servicing ability, I also match on asset turnover and the interest coverage ratio.

Table A3 in the Appendix presents the results of the logit model used to generate the propensity scores. All specifications include year fixed effects. I find that size and growth in sales is negatively related to the probability of receiving treatment (i.e. a PE investment) while the interest coverage ratio is positively associated the likelihood of becoming a buyout target. Asset turnover, somewhat surprisingly, does not have any predictive power in explaining what if a firm is likely to receive buyout investment. Table A5 in the Appendix presents covariate balancing tests to gauge the effectiveness of the match. Column (1) - (4) present univariate tests of differences in means (medians) before matching and columns (5) - (8) present the same statistics after matching. As can be seen, nearest neighbor propensity-score matching greatly reduces any systemic difference between the target and control group.

Second, I carry out an alternate matching criterion to preserve more information and show that the Diff-in-Diff estimates are not sensitive to the type of match. My alternate matching criterion is based on Sales growth and total assets in the same country-year combination. Finally, for each treated company I keep the five closest matched controls to balance the accuracy of matching with the precision of the resulting estimates.

2.2 Descriptive Statistics

Table 1 provides descriptive information for the sample that I use in my determinants analysis. A cursory glance shows there is large dispersion in the data. For example, the median value of total assets for buyout firms is $ 74.1 million while the mean is $ 967. The mean value is similar to Cohn et al. (2014) who use tax return data in U.S. private firms and is significantly higher than the median suggesting there are a large number of small and mid-sized transactions and a few much larger deals. Comparison with other papers using high-quality datasets, though not circulated yet, also confirm that the sample is representative of the traditional leveraged buyout universe. The median (mean) buyout firm has asset tangibility of 0.83 (0.71)[8].

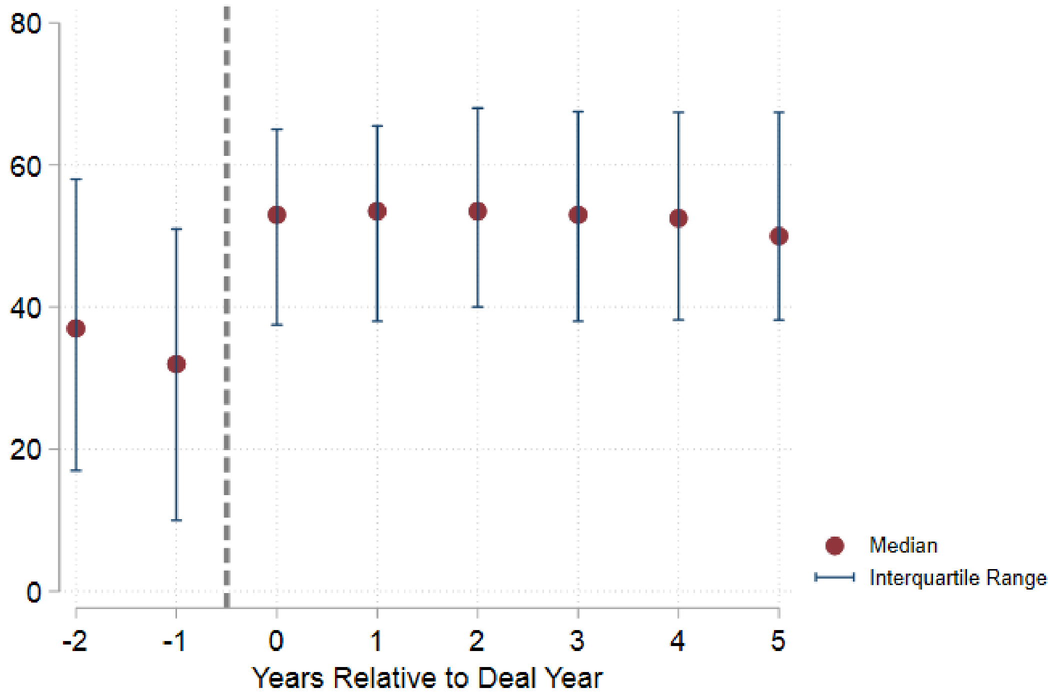
Table 1: Summary Statistics of PE-backed Firms

	Obs	Mean	Std. Dev	p25	p50	p75	p95
Total Assets ($ Mn)	4,864	967	4,670	21.2	74.7	176	4,290
Asset Tangibility	4,439	0.71	0.29	0.52	0.83	0.97	0.99
Profit Margin (%)	4,340	7.72	20.59	0.52	6.51	15.67	42.54
EBITDA Margin (%)	3,917	14.15	19.98	5.07	11.34	21.51	38.43
Quick Ratio	4,355	1.69	3.21	0.66	1.07	1.69	3.03
Cash Assets Ratio	4,325	0.10	0.13	0.01	0.05	0.14	0.38
Sales ($ Mn)	3,544	661	603	17	61	151	2255
Sales Growth	3,165	9.97	51.2	-7.76	3.25	16.17	21.65
Net Debt/EBITDA	3,084	4.95	6.39	1.41	3.58	7.25	25.89
Share of Short-term Debt (%)	4,092	65.8	30.26	41.89	72.9	94	100

I define Sales Growth as the one-year percentage growth in Sales. Buyout firms have median (mean) Sales growth of 3.25 (9.97) percent consistent with the fact that these are typically mature and established companies for whom sales growth is low relative to early-stage companies. Consistent with this fact, buyout firms have EBITDA margins in excess

[8]Note that values from Cohn et al. (2014) and Cohn et al. (2020) are pre-buyout, Table 1 in my sample does not make this distinction.

Figure 1: Trend in Debt Ratio (%) Relative to Deal Year



of 10 percent and positive cash flow. Next, we see that the set of PE firms in the sample are reasonably liquid. Mean (median) Current Ratio is 2.06 (1.37). Mean (median) Liquidity ratio also known as the Quick ratio, defined as $\frac{Current\ Assets\ -Inventory}{Currently\ Liability}$ is reported at 1.67 (1.07). Since both ratios are greater than 1, we can infer that the sample, on average, is not dominated by firms unable to pay their immediate obligations. There is wide variability in the asset turnover ratio. If we consider an asset turnover ratio of 2.0 an indicator of a reasonably efficient firm, more than half the firms fall below this threshold.

There is also high variability in both the debt ratio as well as the leverage ratio. Median debt ratio is 0.62. Adjusted for cash and cash equivalents, this value drops to 0.50. Median leverage ratio is 4.48, which drops to 3.58 once adjusted for cash. While the mean debt ratio is similar to the median, the mean leverage ratio is much higher than the median leverage ratio. A small number of firms have negative values once adjust for cash.

Figure 1 plots the trend in $\frac{Net\ Debt}{Asset}$ around the buyout event. First, the plot reveals that leverage varies significantly across companies. Second, it reveals sizeable increase in debt following buyout. Median debt ratio rises sharply from 32% in $t = -1$ to 53% in $t = 0$, where $t = 0$ is the buyout year. We note that the debt ratio stays elevated within the 50-55% range for several years following the buyout with a small decline in t=5. The inter-quartile range of entry debt ratio is 37% to 65%. Levels and trends in my main outcomes are consistent with Brown (2021) who use deal-level debt data from StepStone and an anonymous global international bank[9].

2.3 Empirical Methodology

To disentangle characteristics unique to PE, I compare the average outcome of acquired (treated) companies with non-acquired (control) companies. The regression analogue of this comparison is a fully dynamic matched difference-in-differences specification outlined in Eq. (1) below. The first difference compares private-equity owned companies before and after acquisition, and the second difference compares target facilities to those that were never private-equity owned. For a difference-in-differences setting with more than two time periods, Imbens and Woolridge (2007) suggest introducing a policy dummy that is simply defined to be unity for groups and time periods subject to the policy along with a full set of time-period dummies. Specifically, I estimate the following generalized Difference-in-Differences:

$$Y_{it} = \beta_1 PE_{it} + \gamma^{'} \mathbf{X}_{it-1} + \alpha_i + \delta_y + \epsilon_{it} \tag{1}$$

For firm i at time (year) t, the dependant variable will alternatively be (i) Net Debt, scaled by assets, (ii) Debt Maturity, defined as the share of long-term debt in total debt expressed in logs (iii) Net Cash Flow, defined as Net Profit plus Depreciation scaled by assets, (iv) Efficiency, proxied by Capital-Labor Ratio, (v) Intangible Assets, scaled by Total

[9]Brown (2021) presents new high-quality propriety data on companies acquired through leveraged buyouts. Due to extensive industry verification, data reported in this paper serves as a reliable benchmark to compare moments in my sample.

Assets and (vi) Risk proxied by Volatility of Earnings Before Interest, Taxes, Depreciation and Amortization, $EBITDA$. PE takes a value of 1 in the years following a leveraged buyout deal. The vector X includes standard controls identified in the literature and varies depending on the outcome of interest. For Net Debt and maturity, I control for standard Rajan and Zingales (1995) variables: $Log\ (Total\ Assets)$, $EBITDA\ margin$, $Tangibility$ and $Sales\ Growth$. For all other outcomes, I control for firm size and leverage. I include firm α_j and year δ_y fixed effects. I cluster standard errors at the firm-level. Our coefficient of interest is β_1, the DiD estimate of PE that provides a causal estimate of PE effect on firm-level outcomes.

2.4 Results

Critics argue that higher leverage creates debt overhang leading to under-investment and raises the likelihood of bankruptcy. On the other hand, Gompers, Kaplan and Mukherlyamov (2015) document that PE managers place heavy emphasis on adding value to their portfolio companies. These view points suggest if PE optimizes higher leverage instead of over-levering companies, we should observe a rise in firm value. In other words their evidence is consistent with traditional finance theory which posits that the relationship between risk and return is positive.

This section discusses results of the matched difference-in-differences on firm-level outcomes. The results are summarized in Table 1. We begin by documenting the fact that debt rises significantly following PE acquisition, controlling for standard capital structure determinants. $\frac{Net\ Debt}{Asset}$ rises by 0.193 relative to the pre-deal sample median of 0.32. In other words, debt rises by at least 60 percent following buyouts. Column (2) documents that debt maturity rises substantially as well. The dependant variable is expressed in natural logs. Using the exponential form of the estimated co-efficient, I find the share of long-term debt in total debt rises by around 65 percent. Since higher debt maturity reduces refinancing risk, one interpretation is that PE mitigates risk of higher leverage by lowering the likelihood of maturity mismatch. This can encourage firms to invest in long-term projects that generate higher cash flows in the future.

Column (3) and (4) provide suggestive evidence of this viewpoint. We note an increase

Table 2: Effect of PE-Ownership on Firm-Level Outcomes

Notes: This table summarizes results of Matched Difference-in-Differences regressions outlined in Eq. (1).

	Debt Ratio	*Debt Maturity*	*Cash Flow*	*Intangibles*	*Efficiency*	*Risk*
PE_{jt}	0.193***	0.542***	0.060**	0.044**	0.197*	-0.024*
	(0.026)	(0.180)	(0.025)	(0.018)	(0.107)	(0.013)
R^2	0.236	0.090	0.091	0.091	0.171	0.017
Firm FE	Y	Y	Y	Y	Y	N
Year FE	Y	Y	Y	Y	Y	Y
Controls	Y	Y	Y	Y	Y	Y
N	1131	1096	1305	1293	1195	1301

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

in cash flows (scaled by asset size). We also see the share of intangible assets increase. As proxy for innovation, research and development, the positive coefficient suggest PE encourages such long-term investment consistent with findings from Lerner et al. (2011). Following Beisinger et al. (2020) I measure capital intensity by $\frac{Capital}{Labor}$. Higher capital intensity indicates improved efficiency within an organization. Column (5) shows a significant rise in capital intensity following buyouts consistent with much of the literature documenting improved operational efficiency. Finally, column (6) documents a reduction in risk measured by the volatility of earnings margin. Since this measure is computed at the firm-level (as opposed to the firm-year level) I drop firm fixed effects. Hence, the more appropriate interpretation is that PE-firms tend to exhibit lower risk following the buyout relative to matched controls. To ensure robustness of these result, I repeat the exercise with an alternate matching criterion and additional controls. These regressions are reported in Appendix B. Finally, to validate the parallel trends assumption I run dynamic difference-in-differences and plot estimated coefficients. The results show parallel trends hold. These are reported in Figure A1 in the Appendix.

Next, we document the tendency of PE funds to inject additional equity into a company when it is in distress. Bernstein et al. (2017) find that PE-backed companies behaved differently than a matched control group during the financial crisis. They estimated equity issuances over assets increased by two percentage points relative to their peers. Because PE groups raise funds that are drawn down and invested over multiple years—commitments that are very rarely abrogated—they may have "deep pockets" during downturns. These capital commitments may allow them to make equity investments in their firms when accessing other sources of equity, or financing in general, is challenging.

To test this hypothesis more generally, as opposed to only during aggregate crisis, I define an indicator variable $Distress$ as follows:

$$Distress_{jt} = \begin{cases} 1 & \text{if } EBIT_{jt} < Interest\ Expense_{jt} \\ 0 & \text{otherwise} \end{cases}$$

where $EBIT$ is Earnings before Interest and Taxes. Using this $Distress$ variable, I estimate Eq. (1) with $Y_{jt} = \frac{Equity}{Asset}$. I introduce an interaction between PE-ownership and $Distress$. A positive interaction term coefficient of $PE_{jt} \times Distress$ is indicative of PE-backed firms receiving additional equity issuances compared to a matched control group when they are pushed into distress.

I present these results in Table 3. Controls include lagged values of firm size, profitability, liquidity and leverage. Column (1) and (2), which use different sets of time-varying controls, finds that $Distress$ is negatively correlated with Equity value. Considering the mechanical accounting relationship between Earnings and Equity this result is not surprising.

More importantly, I find that $PE_{jt} \times Distress$ is positive and statistically significant. Compared to the matched control group, PE-owned firms experience a 2.9 percentage point increase in equity value over asset when their debt obligations excess operating income. One caveat with this set of results is that the DiD estimate is no longer statistically

Table 3: Equity Injection During Distress

$Y_{jt} = Equity/Asset$	(1)	(2)	(3)
$PE_{jt} \times Distress_{jt}$	0.029*	0.029*	0.010
	(0.017)	(0.018)	(0.016)
$Distress_{jt}$	-0.048***	-0.047***	-0.039***
	(0.010)	(0.010)	(0.010)
R^2	0.496	0.497	0.323
Firm FE	N	N	Y
Year FE	Y	Y	Y
Controls	Y	Y	Y
N	763	764	764

Standard errors in parentheses.

* $p < .10$, ** $p < .05$, *** $p < .01$

significant when I add firm fixed effects. Nevertheless, we document suggestive evidence of PE-backed firms receiving additional capital when they run into distress which is also consistent with widely documented news during the ongoing COVID-19 crisis. Overall, this finding is consistent with capital structure theory emphasizing the role of endogenous default: firms default at the asset level that equates the marginal cost of keeping a firm solvent with the marginal cost of declaring bankruptcy.

3 Baseline Model

In this section I outline a model of optimal capital structure of PE-owned firms. A simple starting place would be benchmark models such as Leland (1994) and Leland and Toft (1996). However, several characteristics unique to the PE institutional structure prevent direct application of traditional Leland-type models. First, the agent making capital structure decisions is the private equity fund manager, the GP. Second, however, the PE fund is not liable for LBO debt. Third, the GP's payoffs are different from a traditional equity-holder. I embed these distinctions into a capital structure model and estimate optimal leverage for PE-backed firms under various conditions. To underscore

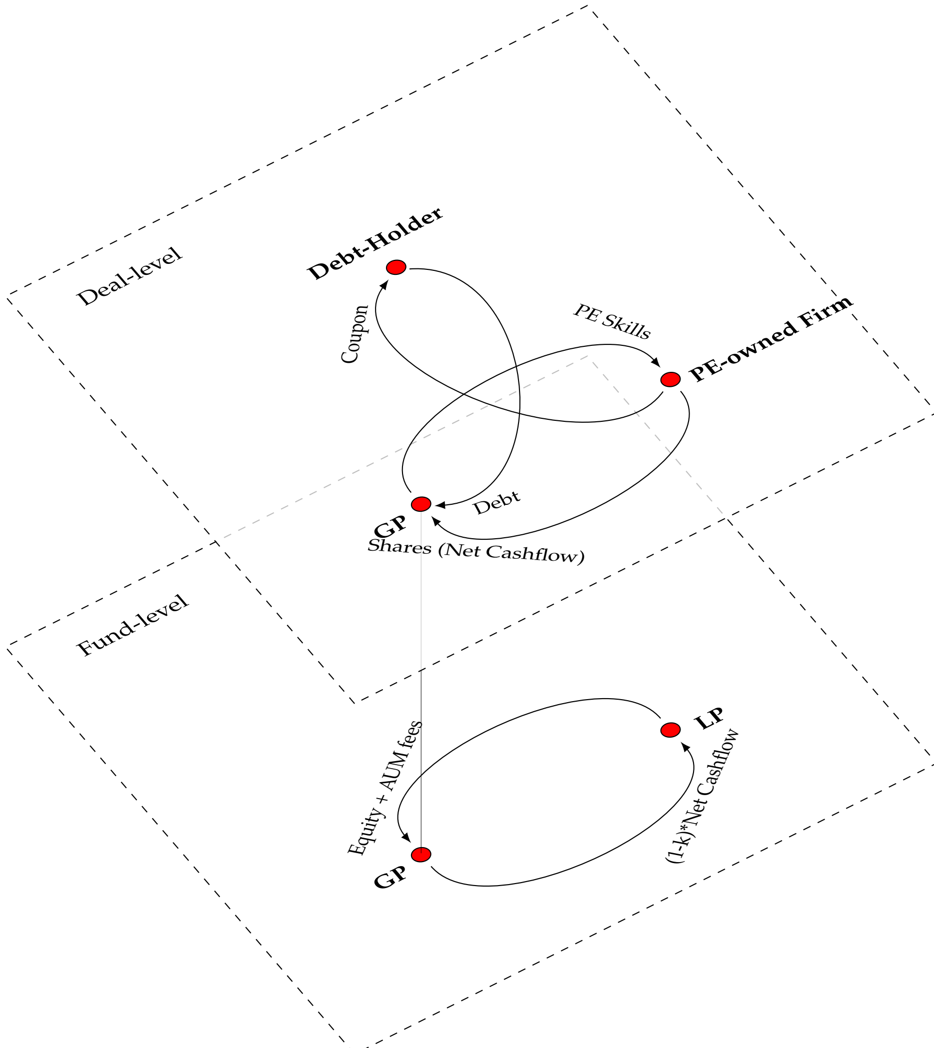
PE's unique characteristics, I benchmark these estimates with model-implied optimal leverage of a non-PE company. Finally, I compare model-implied optimal leverage of both types of companies with the data.

3.1 Environment

This paper models an infinite-horizon economy in continuous time. Markets are complete, and there is a riskless asset that pays a constant rate of interest r per interval of time. Everything is observable implying there is no private information. Following Merton (1974) and Leland (1994), I assume the value of corporate securities depend on underlying firm value but are otherwise time-independent. Time independence allows derivation of closed-form solutions and is usually justified in two ways. First, for debt with sufficiently long horizons the return of principal effectively has no value and can be ignored. Second, time-independence also captures constantly rolled-over debt bearing resemblance to revolving credit facilities (Leland, 1994). In what follows, I capture time-independence at the PE-deal level as well.

All agents are risk-neutral. There is an infinitely-lived agent, called the General Partner (henceforth GP) who can execute private equity investments in companies. Over each time interval $[t, t + dt]$, an investor called a Limited Partner (LP) commits capital worth $I_0(1 + m)$ to the GP, where $m \in (0, 1)$. mI_0 captures an exogenous asset management fee and is discussed below when I outline the GP's compensation. Net of management fees, the GP is left with I_0 initial investment. The GP leverages this amount with D_0 debt to acquire a portfolio company worth $A_0 = D_0 + I_0$. In practice, a GP manages a PE fund and acquires 10-20 portfolio companies, but for tractability I develop my model at the deal-level. It is trivial to show the primary results are identical at the fund-level since a fund's valuation is directly dependant on portfolio company valuations. Within each time interval $[t, t + dt]$, the GP adds value to the company, sells it and receives a profit-sharing

Figure 2: Flows between GP, LP, Debt-Holder and Portfolio Company



fee net of debt obligations and LP commitment[10]. The portfolio company's asset-in-place generates cash flow rate represented by standard Geometric Brownian Motion (GBM) with drift μ_a, volatility σ_a and paying some dividend rate δ:

$$\frac{dA_t}{A_t} = [\mu_a(A,t) - \delta]dt + \sigma_a dB_t^A \tag{2}$$

One interpretation of Eq. (1) is that the firm produces one unit of good per unit of time

[10]An alternate setup with time-independent deals can be summarized as follows: The GP buys and holds company indefinitely, earns management fees plus cash flows net of debt payments and perpetual annuities to LP. This is different from practice but still captures performance-based profit-sharing which is essential to the agent's payoffs.

with market price fluctuating according to GBM. Note also I make the usual assumption of separation of investment and financing policy. Such an approach has intuitive appeal: the cash flow-generating machine, which is the source of firm value, runs independently of how the cash flow is distributed among its claimants. We then define the unlevered value of assets as the expected value of future discounted cash flows that these assets will produce:

$$E_U(A) = \mathbf{E}\Big(\int_t^{\infty} e^{-r(s-t)}(1-\tau)A_s ds\Big) = (1-\tau)\frac{A}{r-\mu} \tag{3}$$

The GP's value-creation role can be in the form of higher revenue growth, improved efficiency, better governance etc. I capture this through an excess return, α, that allows PE-backed companies to growth faster than benchmark non-PE companies. Formally, under appropriate risk-neutral probability measure outlined in Sorenson et al. (2014), I assume:

$$\mu_{PE} = \alpha + \mu_{Non-PE} \tag{4}$$

Figure 2 highlights these unique characteristics while distinguishing between activities at the deal-level and at the fund-level. PE-skills are flows from the GP to the company and results in an α. Net cashflows are cash flows net of interest payments. Observe that bond-holders are being repaid by the portfolio company while the debt is initially raised by the GP.

3.2 Debt-Holders

Over each time interval $[t, t+dt]$ the firm is servicing its debt holders by coupon at the rate C. Now consider any claim on the PE-firm that perpetually pays a non-negative coupon, C, per interval of time when the firm is solvent. If $A_t < C_t$, the company is in distress, and has to raise money externally by either issuing debt or equity; if the company cannot get external financing or chooses not to, it defaults. Debt allows companies to exploit tax-shields, but it also comes with non-negligible costs. Higher leverage increases the probability of default, and makes further debt issuance costlier. When choosing the optimal debt policies, companies trade off these costs and benefits. Similar trade-off theory argument applies to a PE fund manager as well since the GP's payoffs are also dependant on portfolio company performance. This will be discussed in Section 2.3.

As Leland (1994) and He (2014) outline, under the assumption of time-independence of coupon payouts, the valuation or HJB equation for debt-holders can be reduced to the following ordinary differential equation:

$$rV(a) = C + \mu AV^{'}(a) + \frac{1}{2}\sigma^2 A^2 V^{''}(a) \tag{5}$$

with general solution taking the form:

$$V(a) = K_0 + K_\gamma A^{-\gamma} + K_\eta A^{-\eta} \tag{6}$$

where the coefficients are determined by boundary conditions. The two power parameters are roots to the fundamental quadratic equations:

$$\frac{1}{2}\sigma^2 x^2 + (\mu - \frac{1}{2}\sigma^2)x - r = 0 \tag{7}$$

where we assume $V(a) = a^x$, implying $V^{'}(a) = xa^{x-1}, V^{''}(a) = x(x-1)a^{x-2}$. For debt, the flow payoff can be expressed as:

$$D(A) = \frac{C}{r} + K_\gamma A^{-\gamma} + K_\eta A^{\eta} \tag{8}$$

Let ρ represent the fraction of asset value A_B which is lost in the event of bankruptcy. There are two boundary conditions. When $A = \infty$, default never occurs so $D = \frac{C}{r}$ perpetuity. Hence $K_\eta = 0$ otherwise debt value goes to infinity. Absolute priority rule applies, and debt-holder get the value of company's assets if the firm declares bankruptcy. When $A = A_B$, debt value is $\frac{(1-\rho)A_B}{r-\mu_a}$.[11] Substituting into Eq. (6) and solving for K_γ:

$$K_\gamma = \frac{\frac{(1-\rho)A_B}{r-\mu_a} - \frac{C}{r}}{A_B^{-\gamma}} \tag{9}$$

We can then derive closed-form analytical solution for debt value:

$$D(A) = (\frac{A}{A_B})^{-\gamma}\frac{(1-\rho)A_B}{r-\mu_a} + (1 - (\frac{A}{A_B})^{-\gamma})\frac{C}{r} \tag{10}$$

where $\gamma = (r - \delta - 0.5\sigma^2 + [(r - \delta - 0.5\sigma^2)^2 + 2\sigma^2 r]^{0.5})/\sigma^2$.

Observe that while Eq. (8) is convenient for estimation purposes, it can also be expressed in the following intuitive form:

$$D(A) = \mathbf{E}\Big[\int_0^{\tau_B} e^{-rs}C ds + e^{-r\tau_B}\frac{(1-\rho)A_B}{r-\mu}\Big] \tag{11}$$

$$= \mathbf{E}\Big[\frac{C}{r}(1 - e^{-r\tau_B}) + e^{-r\tau_B}\frac{(1-\rho)A_B}{r-\mu}\Big] \tag{12}$$

Eq. (10) expresses payoff to debt holders as a function of the likelihood of solvency and default and respective payouts in each states of the world. As He (2014) and Leland (1994) outline, τ_B is the first passage of time when cash flows fall below the bankruptcy-triggering level.

3.3 Equity-holder (GP)

In traditional capital structure models the equity-holder's payoff resemble a plain vanilla European call option in the sense that claimants receive 0 in the event of bankruptcy and cannot have negative equity. However, the GP receives a management fee which is senior in nature and is invariant to default probability. Additionally, the GP receives a share of

[11] The denominator follows from standard Gordon Growth formula.

profits from the exit price net of debt obligations and LP commitments. Hence, the GP's compensation can be outlined as follows:

$$F(A;A_B;C) = \underbrace{M(I_0;m)}_{Management\ Fee} + \underbrace{I(A_t, C, A_B)}_{Incentive\ Fee} \tag{13}$$

$$= \frac{mI_0}{r} + max(\underbrace{k\{A_t - (1-\tau)C - (1+h)I_0\}}_{Profit\ Sharing}, 0) \tag{14}$$

$$= \frac{mI_0}{r} + k \times Call(A_t, \alpha, C, A_B) \tag{15}$$

Recall management fee is an exogenous annual rate that pays a constant fraction of capital received from investors, mI_0 (e.g. m=2.0%), implying the GP's objective function will involve maximizing incentive fees only. Following Sorenson, Wang and Yang (2018), I model the GP's incentive fee as a claim on the underling portfolio company[12]. At the end of the time interval when the portfolio company is sold, the GP's incentive fee resembles a European call option shown in Eq. (12). k is a fraction of profits the GP will receive assuming $A_t - (1-\tau)C - (1+h)I_0 > 0$, (e.g. k=20%). Since k is exogenous, for simplicity I set it equal to 1 without loss of generality. Note that if a GP injects additional equity to prevent default assuming it is optimal, I_0 will be replayed by $I_{t+dt} = I_0 + \sum_{t=t}^{t+dt} I_t$. [13]

The embedded option in this payoff structure has two immediate implications: (i) Despite not being directly liable for LBO debt, the risk-netural fund manager has incentives to maximize the portfolio company's going concern so as to collect performance fees at time T; The risk-neutral manager is averse to bankruptcy and this precautionary motive induces risk-averse managerial behavior. (ii) Alternatively, funds may be tempted

[12]Sorenson, Wang and Yang also include a positive hurdle rate and catch-up region. For simplicity I only model the hurdle rate and retain the critical relationship of seniority between creditors, LP and the GP since debt holders have to be paid first. Then the LP has to be returned the committed capital before any profit-sharing can occur.

[13]In reality PE funds are close-end so additional equity injection is only possible if I allow for uncalled capital. Since uncalled capital is merely additional equity that was not invested but supplied by LP, it will not affect the model setup.

to take excessive risk when they are compensated via incentive fees. Axelson, Stromberg, and Weisbach (2009) present a model in which fund managers tend to over-invest, taking value-decreasing investments in addition to value-increasing ones because of their option-like compensation. The compensation structure outlined above captures a similar incentive. Risk-shifting and related agency costs will be captured through the estimated asset volatility parameter. Thus which implication is consistent with the data is ultimately an empirical question.

Following standard derivation steps for the Black-Scholes-Merton PDE and assuming time-independence (i.e. $\frac{\partial E}{\partial t} = 0$), the GP's HJB equation reduces to the following ODE that equates the required return to a flow payoff and local change of value function (capital gain, long-term payoffs):

$$rE(a) = mI_0 + f(a) + \mu AE^{'}(a) + \frac{1}{2}\sigma^2 A^2 E^{''}(a) \tag{16}$$

Eq. (16) that values the GP's interest is different from the standard Black-Scholes-Merton PDE due to the term $mI_0 + f(a)$, which represents the GP's inflow of ongoing asset management fees and standard flow payoff as the equity-holder of the portfolio company. It is well-known that the general solution to $E(a)$ is given by the following expression:

$$E(a) = \frac{mI_0}{r} + \frac{A}{r-\mu} - \frac{z}{r} + K_\gamma A^{-\gamma} + K_\eta A^\eta \tag{17}$$

where strike price $z = (1-\tau)C + (1+h)I_0$

The first boundary condition is when $A = \infty$. Equity value cannot grow faster than first best firm value which is linear in A, meaning $K_\eta = 0$. The second boundary condition is when $A = A_B$; GP still receives asset management fees $\frac{mI_0}{r}$, which are senior in nature and resemble a risk-free annuity per interval of time.

$$E(a) = \frac{mI_0}{r} + \frac{A}{r-\mu} - \frac{z}{r} + K_\gamma A^{-\gamma} + K_\eta A^\eta = \frac{mI_0}{r} \tag{18}$$

which simplies to:

$$E(a) = \frac{A}{r-\mu} - \frac{z}{r} + K_\gamma A^{-\gamma} + K_\eta A^\eta = 0 \tag{19}$$

Using boundary conditions and rearranging Eq. (14) yields:

$$K_\gamma = \frac{\frac{z}{r} - \frac{A_B}{r-\mu}}{A_B^{-\gamma}} \tag{20}$$

$$E(A) = \frac{A}{r-\mu} - \frac{z}{r} + (\frac{z}{r} - \frac{A_B}{r-\mu})(\frac{A}{A_B})^{-\gamma} \tag{21}$$

Substituting the expression for K_γ back into the Black-Scholes-Merton ODE gives Eq. (16). Using smooth-pasting condition I solve for the bankruptcy-triggering cash flow level A_B shown in Eq. (18):

$$h(c, A_B) = \frac{\partial E(A, A_B)}{\partial A}\Big|_{A=A_B} = 0 \tag{22}$$

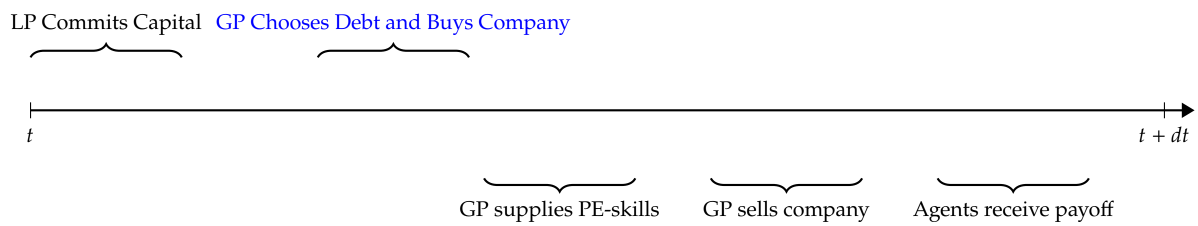
$$A_B = ((1-\tau)C + ((1+h)I_0)\frac{r-\mu}{r}\frac{\gamma}{1+\gamma} \tag{23}$$

The bankruptcy-triggering cash flow level differs from traditional capital structure models through the $I_0(1+h)$ term that the GP must return to LP. An immediate implication is that higher hurdle rate or pricing in additional equity issuances will raise the bankruptcy-triggering cash flow level and, by construction, lower optimal leverage. The following comparative statics predict changes in bankruptcy-triggering cash flow level with respect to shifts in variables in interest. Note that a higher (lower) bankruptcy-trigger will decrease (increase) optimal leverage.

$$\frac{\partial A_B}{\partial r} > 0, \frac{\partial A_B}{\partial h} > 0, \frac{\partial A_B}{\partial I_0} > 0, \frac{\partial A_B}{\partial \mu} < 0, \frac{\partial A_B}{\partial \alpha} < 0, \frac{\partial A_B}{\partial C} > 0 \tag{24}$$

The partial derivatives in Eq. (21) shows an increase in risk-free rates raise the

Figure 3: Timeline of the Model



bankruptcy-trigger and by extension lowers optimal leverage. This is different from Leland (1994) that predicted the increase in expected default costs was completely offset by higher tax shield benefits and warranted a higher optimal leverage. Not surprisingly, we find higher hurdle rates and initial capital commitments also raise the bankruptcy-trigger. Perhaps most importantly for the research question in this paper, we note higher drift rates cause the bankruptcy-triggering level to decline. Plugging in Eq. (3) by substituting out μ shows higher α raises optimal leverage. Since PE delivers an excess return, the partial suggests higher excess returns will justify higher leverage. Finally, we verify that higher coupon payments reduce optimal leverage similar previous papers. I end this section by deriving a closed-form expression for equity value and the GP's payoff by plugging A_B back into the equity-holder's HJB.

3.4 Levered Firm Value and GP's Optimization Problem

Figure 3 summarizes the timeline of the model for each time interval $[t, t + dt]$. Since the GP is maximizing expected payoffs from incentive fees, the optimization problem reduces to maximizing value of the underlying firm under the assumption that incentive fees are increasing in A_t. The optimization problem can then be conceptualized in two steps. First, determine the optimal A_B by maximizing the equity value of the portfolio company, covered in the previous section. Second, determine the optimal leverage by maximizing the value of the levered company. This second step is equivalent to solving for the optimal coupon. Much of the literature on capital structure literature derives total value of the levered firm as the sum of the firm's unlevered value, tax benefits and bankruptcy costs (eg: Leland (1994) and Leland and Toft (1996), Goldstein et al. (2001)). Tax benefit represents a flow payment representing tax shields each time interval with boundary conditions similar to those shown for debt and equity holders. Bankruptcy cost is also determined at the boundary when $A = A_B$

$$v(A) = E_U(A) + TB(A) - BC(A) = \underbrace{\frac{A}{r-\mu}}_{Unlevered\ Value} + \underbrace{\frac{\tau C}{r}[1-(\frac{A}{A_B})^{-\gamma}]}_{Tax\ Shield} - \underbrace{\frac{\alpha}{r-\mu}A_B(\frac{A}{A_B})^{-\gamma}}_{Distress\ Cost} \tag{25}$$

Formally, the firm, managed by the GP, solves the following problem:

$$\underset{c.A_B}{max}\ I(A_0, c, A_B) \equiv \underset{c,A_B}{max}\ v(A, A_B, c)\Big|_{A=A_0} \tag{26}$$

subject to the bankruptcy-triggering condition:

$$h(c, A_B) = 0 \tag{27}$$

I make standard assumptions on $v(C)$: $v^{'}(c) > 0$, $v^{''}(c) < 0$ and $v^{'}(c^{*}) = 0$ for some finite $c^{*} > 0$. Maximising Eq. (24) subject to Eq. (25) and solving for the optimal coupon gives us the following expression:

$$C^{*} = \frac{A_0}{r-\mu}\frac{r(1+\gamma)}{(1-\tau)\gamma}\big((1+\gamma) + (1+\rho)\gamma\frac{(1-\tau)}{\tau}\big)^{-\frac{1}{\gamma}} \tag{28}$$

Substituting Eq. (24) back into (9) and (21) will yield expressions for optimal debt and firm value. In addition to standard sensitivity of coupon to tax rates and firm risk documented in previous studies, note that optimal coupon will be affected by PE's excess return generation capacity which will be captured through $\mu_{PE} > \mu_{Non-PE}$. In the baseline case, I allow endogenous bankruptcy justified on grounds mentioned earlier. Since it is also possible a fund is completely invested when a company enters distress, endogenous bankruptcy may not be possible. Positive net-worth type covenants are quite common in PE. For example, Achlietner et al. (2011) explore covenant structures in LBOs that are based on $\frac{Debt}{EBITDA}$ where $EBITDA$ = Earnings before Interest, Taxes, Depreciation and Amortization. To minimize computational cost, I choose the Interest Coverage Ratio as the variable to use as an exogenous default threshold. As an alternate bankruptcy condition I introduce a positive net-worth type covenant structure based on $\frac{EBIT}{Interest\ Expense}$. This condition can be expressed as $A_B = \frac{C}{A} = d$ where $d \in (0, 1)$ is an exogenous constant. One approach would be to force the firm to default whenever $C = A$. Since we documented in Section 2 neither type of firm declare bankruptcy if earnings fall below required interest expense, I choose $d < 1$ considering firms could still use internal cash to repay debt, but do not have sufficient resources for endogenous default. This can also be justified on the grounds that firms usually save on precautionary motives to hedge against bad states of the world.

4 Model Results

4.1 Model-Implied Optimal Leverage

This section presents the quantitative results of the model. First, I set initial asset value in each firm-year to 100 and scale other variables to this value when necessary. I estimate certain parameters such as required rate of return and firm risk or asset volatility separately for PE-company and control firms separately. Most importantly, following Bartram et al. (2013) I specify:

$$\sigma_a = exp(\beta_0 + \sum^{n} \beta_i X_i) \tag{29}$$

where X_i is a set of covariates including firm size, tangibility, profitability, liquidity and profit volatility; β_i are the estimated coefficients using maximum likelihood[14]. I choose an exponential function to ensure positive values for σ_a.

Table 4: Parameter Values

Parameter	Value	Source
Common Across Firm-Type		
τ	0.30	LT (1996)
r	0.05	10-year U.S. Treasury
δ	0.01	Leland (1994)
ρ	0.25	LT (1996)
μ	0.02	Standard
PE Firm		
α	0.01	SWY (2014)
σ	0.25	Estimated with Eq. (23)
d	5.00	Standard
Non-PE/Benchmark Firm		
α	0.00	-
σ	0.29	Estimated with Eq. (23)
d	3.00	-

I consider the parameter values outlined above as my baseline values. For each parameter, I will re-estimate the model keeping all but one parameter as in the benchmark set. Of particular interest is understanding how the outcome changes with respect to (i) PE's excess return (ii) Firm Risk (iii) Risk-Free rates.

My dataset is global and spans a large number of mostly advanced countries. Since risk-free rates is most advanced economies are closely linked to the U.S. risk-free rate, I use the U.S. long-term treasury rate. Regarding the set of parameters that are calibrated outside of the model, I set $d = 0.3$ meaning companies will default if their interest coverage ratio falls to 0.3 in the exogenous bankruptcy case.

Table 4 summarizes the main results of the estimated model. Row 1 estimate optimal leverage ($\frac{Debt}{Asset\ Value}$) for a Non-PE owned company that serves as a benchmark to understand the dynamics of PE-backed firms. There are three differences between PE and

[14]These estimates are available on request. I find all variables significantly explain asset volatility.

Non-PE. First, Non-PE firms always default according to the exogenous default threshold whenever $\frac{EBIT}{Interest\ Expense} < 0.3$. Second they do not earn any $alpha$, thus I set $alpha = 0$, and their risk level σ is different. I estimate σ using firm-specific data using Eq. (??). I normalize all firm values in subsequent rows to the benchmark company's firm value estimated at its optimal leverage. I find optimal leverage of benchmark companies to be 37.9 percent, broadly consist with observed leverage of most U.S. public firms and historical ratios documented in the literature.

Row 2 presents results of the model for the benchmark set of parameters and, in subsequent rows, for some variation of the parameters. PE-backed firms have much higher optimal leverage of 59.9 percent. The potential gains in optimizing financial structure are considerable. For the baseline set of parameters, firm value modelled as Earnings Before Interest and Taxes can increase by as much as 55.6 percent when an optimally levered non-PE backed company is acquired by PE. Row 3 shows optimal leverage rises substantially if we increase excess return to 2 $percent$. Since this is a perpetual excess return, the rise in firm value is much more significant. In Row 4, I set $\alpha = 0$, and find that even without any excess return optimal leverage in PE is close to 50 percent. The value generation is primarily coming from endogenous bankruptcy and marginally lower cash flow volatility. However, without excess return, the gain in firm value is very small compared to the benchmark case.

Row 5 and 6 illustrate differences in optimal leverage from variations in risk. We note that relative to the baseline case, 10 percentage point change in risk does not affect optimal leverage and firm value considerably in the endogenous bankruptcy case. Finally, row 6 shows optimal leverage is much higher if PE firms did not price in the hurdle rate that LPs demand. Next, we see how changes in market factors affect optimal leverage. Row 7 shows an increase in risk-free rate by 1.8 percent[15]. Optimal leverage decreases by about 10 percentage point compared to the baseline due to higher expected bankruptcy

[15] This represents a 1 standard-deviation shock in interest rates in Chari et al. (2020).

costs. Finally, a 5 percent positive shock to tax rates raises optimal leverage by about 2.5 percentage points relative to the baseline due to higher tax shields.

Parameter Uncertainty: In the next step, I introduce parameter uncertainty into key model ingredients. While the static results reported in Table 5 suggest baseline optimal leverage is broadly consistent with the data, it is possible possible average values in a cross-section differ markedly from what firms choose at T_0. To examine the sensitivity of the model, I simulate a cross-section of PE-firms with heterogeneity in α and σ. To this end, I generate an economy populated by $N = 2000$ PE-backed firms that initiate activities outlined in Figure 2. For each firm i, excess return and risk are characterized as follows:

$$\alpha_i = 0.02 + \epsilon, \ \epsilon \sim \mathcal{N}(\mu_a, \sigma_a) \tag{30}$$

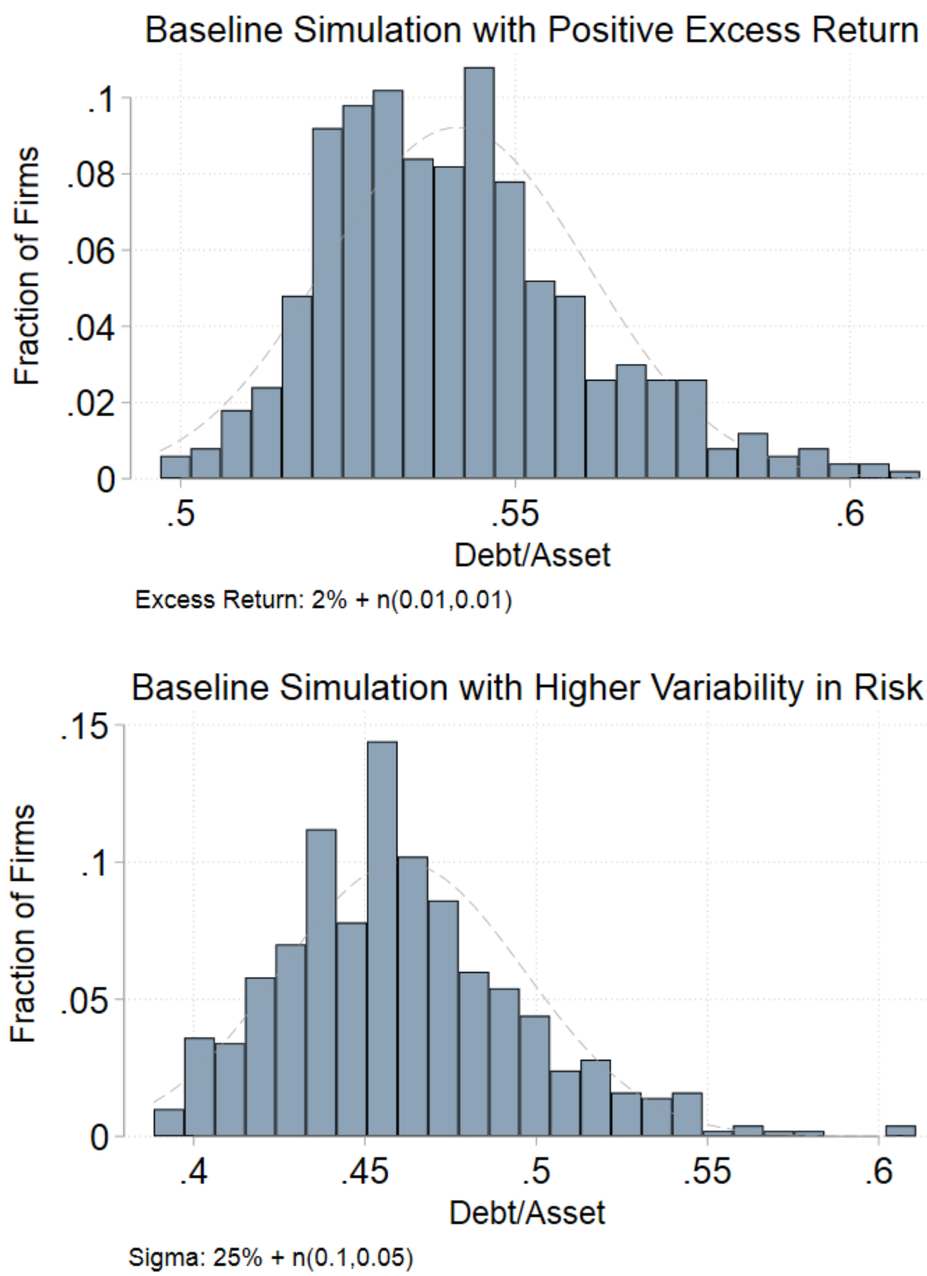
$$\sigma_i = 0.25 + \zeta, \ \zeta \sim \mathcal{N}(\mu_\sigma, \sigma_\sigma) \tag{31}$$

I set $\mu_a = 0.01$, $\sigma_a = 0.01$, $\sigma_\sigma = 0.1$ and $\sigma_\sigma = 0.05$. I re-estimate optimal leverage for each firm and plot the distribution of optimal leverage in Figure 4. The introduction in heterogeneity in excess return reduces optimal leverage to marginally below 55 percent for the largest share of firms. However, the higher variability in risk reduces optimal leverage much more to around 45 percent as shown in the bottom panel of Figure 4.

Table 5: Model Results

	Optimal Leverage (%)	Firm Value (Normalized)
Non-PE Company (Benchmark)	37.9	100.0
PE-backed Company		
Baseline	59.9	155.6
$\alpha = 2\%$	70.1	321.6
$\alpha = 0\%$	49.9	103.2
$\sigma = 0.15$	62.5	161.7
$\sigma = 0.35$	58.4	153.6
$h = 0$	68.7	165.2
$Interest\ Rate\ Shock : r + 1.8\%$	50.9	80.3
$Tax\ Rate\ Shock : \tau + 5\%$	62.5	160.4

Figure 4: Risk-Return and Model-Implied Leverage Ratio



4.2 Cost of Deviating from Optimal Leverage

Since much of the criticism of PE centers around high leverage ratios, a natural counterfactual experiment is quantifying the loss in firm value from deviating from optimal leverage. In this section, I ask how much value is lost if PE-backed companies remained at leverage ratios similar to non-PE companies? Note, this question implicitly assumes all other characteristics of PE-backed companies still exist: endogenous bankruptcy, higher excess return, estimated risk from firm-specific data.

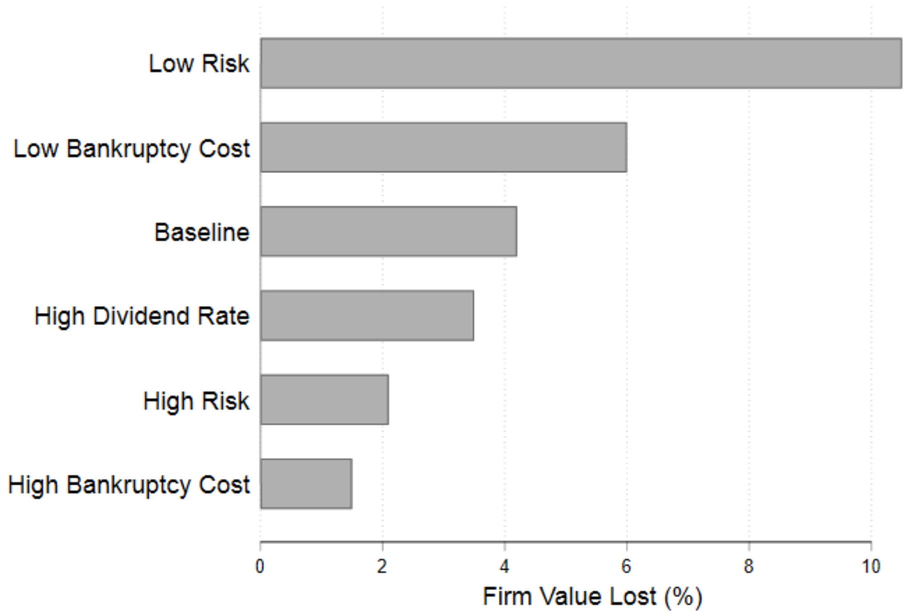
To answer this question, I estimate firm value $V^{'}$ if PE-backed firms levered up to the optimal leverage ratio of benchmark/non-PE companies reported in Table 5 (38%). Letting, V^{*} denote firm value at PE's optimal leverage ratio reported in Table 5, I compute $Cost\ of\ Deviation = V^{*} - V^{'}$. Next, I repeat the exercise for the following cases: (i) high-risk PE company, (ii) low-risk PE company, (iii) high bankruptcy cost, (iv) low bankruptcy cost and (v) high payout rate. Parameter values for each of these cases are reported in Table 6.

Table 6: Parameter Values

	Parameter	Calibrated Values
Low Risk	σ	0.05
High Risk	σ	0.35
High Payout Ratio	d	0.025
High Bankruptcy Cost	ρ	0.5
Low Bankruptcy Cost	ρ	0.05
Baseline	–	See Table 4

The results are plotted in Figure 5. I find that cost of remaining at sub-optimally low leverage ratios is most severe for low risk PE-backed firms. It is worth mentioning that the typical target of a levered buyout are large companies with stable cashflows (low σ). Low-risk PE backed firms stand to lose as much as 10.5 percent of firm value if they did not lever up to their optimal levels. I also observe cost of deviation is quite high when bankruptcy costs are low (6 %), followed by the baseline case (4.2%). However, the cost of

Figure 5: Cost of Remaining at Benchmark/Non-PE Company Leverage



levering up is low if the company is characterized by high risk or if bankruptcy costs are high. Since optimal leverage ratio is much lower relative to the baseline when risk is high (or when bankruptcy cost is high), these results are consistent with much of the capital structure literature (e.g. Leland (1994), Goldstein et al. (2001).

4.3 Probability of Default

In this section I outline the procedure I follow to estimate distance to default and probability of default. It is worth noting that various implementations of the Merton (1974) model remain by far the most common structural method for measuring credit risk, both in the academic literature and in industry (e.g., Moody's/KMV). Given that more flexible and realistic capital structure models exist, such as those suggested by Leland (1994a,b) and Leland and Toft (1996), this may appear somewhat surprising. Forssbæck and Vilhelmsson (2017) suggest the likely reason lies in the difficult of estimating parameters of leland-type models.

Thus, I begin by estimating the canonical model in Merton (1974) as a benchmark. I contrast implied default probability from Merton (1974) with the same from my model that embeds endogenous bankruptcy and a GP's compensation structure. As already mentioned, since the equity stake of a firm can be seen as the residual claim on the firm's assets after debt has been repaid, the Black and Scholes (1973) option pricing formula can be used to calculate the equity value according to:

$$E = N(d_1)A + De^{-rT}N(d_2) \tag{32}$$

where $d_1 = \frac{\frac{A}{D}+(r+0.5\sigma_A^2 T)}{\sigma_A\sqrt{T}}$ and $d_2 = d_1 - \sigma_A\sqrt{T}$. As is well-known in the credit risk literature, D is the firm's debt, all of which is assumed to mature at the same time T, $N(\cdot)$ the cumulative distribution function of the standard normal under the risk neutral measure.

The default probability P_{def} is the probability that the firm's assets will be less than the book value of the firm's liabilities. In other words:

$$P_{def,t} = Prob(V_{A,t+T} \leq D_t|V_{A,t}) = Prob(ln(V_{A,t+T}) \leq ln(D_t|V_{A,t}) \tag{33}$$

As Vassalou and Xing (2004) outline, we can then express the value of assets based on standard GBM process:

$$ln(V_{A,t+T}) = ln(V_{A,t}) + (\mu - \frac{\sigma_A^2}{2}T + \sigma_A\sqrt{T}\epsilon_{t+T}) \tag{34}$$

where $\epsilon_{t+T} \approx N(0,1)$. Let $V_{A,t} = A_t$, we can then express the distance-to-default (DD) as follows:

$$DD = \frac{\frac{A_t}{D} + (u_A + 0.5\sigma_A^2)T}{\sigma_A\sqrt{T}} \tag{35}$$

where u_A is used under the physical probability measure. I link equity values to asset values using the following expressions:

$$\sigma_E = \frac{A}{D} N(d_1) \sigma_A \tag{36}$$

This generates σ_A and A by simultaneously solving Eq. (??) and Eq. (??). Equity volatility is estimated as the standard deviation of equity values for each company. I set the default barrier as:

$$D = Short\ Term\ Debt + 0.5 * Long\ Term\ Debt \tag{37}$$

I follow an iterative estimation method for the Merton model proposed by Vassalou and Xing (2004), and also employed by, e.g., Bharath and Shumway (2008) and is conceptually similar to the maximum likelihood estimation procedure of Duan (1994, 2000). The iterative algorithm starts by guessing an initial value for σ_A , which I set equal to $\sigma_A^0 = \sigma_E \frac{\sigma_E}{E+D}$, where E is the book value of equity at the end of the year, and D is the book value of debt as defined earlier. I set $r = 1.5\%$. Given these estimates and initial values, I can solve Eq. (??) for A to obtain the first iteration. I repeat this exercise until I achieve a standard convergence criterion. Defining $\epsilon < 0.001$, I stop the algorithm when:

$$|\sigma_A^n - \sigma_A^{n-1} < \epsilon| \tag{38}$$

Next, I estimate expected default probability based on my model. DD is calculated as follows in this case:

$$DD = \frac{\frac{A_t}{A_B} + (\mu_A - \delta + 0.5\sigma_A^2)T}{\sigma_A \sqrt{T}} \tag{39}$$

Following Vassalou and Xing (2004) and Bharath and Shumway (2008), default probability for each model is given by:

$$P_{def}^{Merton} = N(-DD) = N(-\frac{\frac{A_t}{D} + (\mu_A + 0.5\sigma_A^2)T}{\sigma_A \sqrt{T}}) \tag{40}$$

$$P_{def}^{Baseline} = N(-DD) = N(-\frac{\frac{A_t}{A_B} + (\mu_A - \delta + 0.5\sigma_A^2)T}{\sigma_A\sqrt{T}}) \tag{41}$$

Observe the difference between Eq. (38) and Eq. (39). In Merton (1974), asset value is scaled by the face value of debt, whereas, in my baseline model the corresponding variable is the endogenous bankruptcy-triggering asset level. Note that the tax benefits and cost of default is embedded in A_B since it is derived from the difference between the value of the levered firm and debt (e.g. see Leland and Toft (1996)). Additionally, firms in the baseline model also have a non-zero payout rate. To estimate Eq. (39) I set the endogenous default-barrier as the solution to Eq. (??). The firm defaults if Earnings before Interest and Taxes falls below the endogenous threshold[16]. Since the primary characteristics of the baseline model only exist post-buyout, I only estimate it for $t \geq 0$ where t is the deal year.

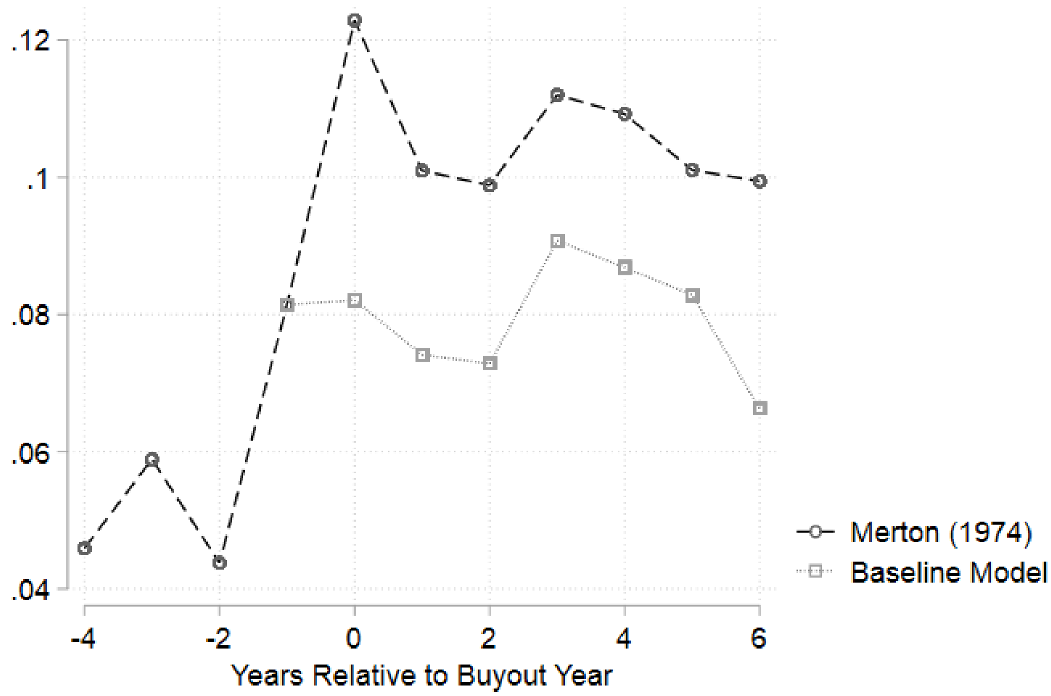
I plot these estimated default probabilities for each year relative to the deal year in Figure 6. To mimimize the influence of outliers, I winsorize default probability at the 0.1 percent and 99 percent level[17]. We observe a striking divergence in default probability: the Merton (1974) model predicts much higher default probability relative to my baseline model post-LBO. In the year of the buyout, the increase in debt raises default probability from 8 percent to above 12 percent in Merton (1974), whereas it hardly changes in the baseline model. The fact that the baseline model predicts lower default probability on average is not surprising and highlights the role of PE's "deep-pockets" in suppressing default probability. In unreported estiamtes, I confirm that the payout ratio is not the primary driver of this striking difference.

Next, I summarize key moments in Table 7, Panel A. The moments are computed for the same time-frame as that shown in Figure 5. First, pre-LBO default probability is relatively low for PE-backed firms pre-LBO with mean and median values of 5.7 and 5.2

[16]An alternate approach is proposed by Korteweg and Polson (2009) which is also explored as robustness. These are available on request.

[17]There is a large cross-section of firms with default probability close to 0, thus the median default probability is negligible.

Figure 6: Divergence in Probability of Default: Merton (1974) vs. Baseline Model



percent respectively. Post-LBO, the merton Model predicts an almost doubling of default probability. We see mean default probability rises by by 4.9 percent to 10.6 percent while the median is at 10.1 percent. However, when estimated using the baseline model, mean default probability is much lower at 7.9 percent. The median is only marginally higher.

I repeat the exercise in Panel B, but with $r = 0.05$ reflecting the ultra-low interest rate environment characterizing much of the post-global financial crisis period. Axelson et al. (2013) argue buyout leverage is primarily driven by “cheap debt”, which in turn should increase default probabilities if a GP chooses capital structure without internalizing company fundamentals. However, I observe default probabilities are lower for both models, with Merton (1974) predicting higher bankruptcy likelihood. These estimates underscore the necessity of capturing PE’s “deep-pockets” through optimal bankruptcy conditions and excess return. Without accounting for these key features, it is not surprising that default probability is likely to be over-estimated.

Table 7: Estimated Moments of Default Probability (%)

	Mean	Median
Panel A: Main Results		
Merton (1974)		
Pre-LBO	5.7	5.2
Post-LBO	10.6	10.1
Change in Default Probability	4.9	4.9
Baseline Model		
Pre-LBO	-	-
Post-LBO	7.9	8.2
Change in Default Probability	3.0	3.3
Panel B: Results with ultra-low interest rates ($r = 0.5\%$**)**		
Merton (1974)		
Pre-LBO	5.8	5.1
Post-LBO	8.6	8.5
Change in Default Probability	2.8	3.4
Baseline Model		
Pre-LBO	-	-
Post-LBO	7.1	7.2
Change in Default Probability	1.3	2.1

5 Conclusion

Private Equity is widely criticised for putting on too much debt on their portfolio companies' balance sheets. Critiques often cite cases such as Toys R Us to support their claim. This standard criticism implicitly assumes what optimal leverage should be for a company. However, optimal leverage is unobservable without a structural model that endogenizes key benefits and costs of debt as well as the incentives governing the agent choosing capital structure, the GP. This paper, is the first to show using a structural model, that private equity ownership can lead to higher levels of optimal debt.

I beging by establishing two stylized facts using a uniquely constructed dataset of PE deals with pre and post-buyout company financials. Using a set of propensity-score matched difference-in-differences, I show that PE-backed companies generate higher cash flows and receive equity injection when in distress. These findings motivate the need to model an excess return and optimal bankruptcy-triggering level in a PE-model of capital structure.

Next, I introduce a novel structural model that embeds a GP's payoffs and estimate optimal leverage for PE-owned companies and compare it with the data. The results indicate PE-firms are not systematically over-levered. Rather higher observed leverage is optimal as long as PE firms can generate an α and choose default-triggering asset level optimally. Computing default probabilities and bench-marking with traditional credit risk models shows that PE firms have lower default probability when we take into account its unique characteristics. These findings are novel in both the PE literature and the broader Leland-type capital structure literature.

References

[1] Aldetmaz and Brown, 2019, Private Equity in the Global Economy: Evidence on Industry Spillovers, Working Paper

[2] Aldatmaz, Brown and Demirgüç-Kunt, Determinants of International Buyout Investments, 2020, World Bank Policy Research Paper

[3] Asis, Gonzalo, Anusha Chari and Adam Haas, 2020, In Search of Distress Risk in Emerging Markets, NBER Working Paper 27213

[4] Brown, Greg, 2021, Debt and Leverage in Private Equity: A Survey of Existing Results and New Findings, Institute for Private Capital, Working Paper, Retrieved from University of North Carolina at Carolina at Chapel Hill, Institute for Private Capital

[5] Bartram, Sohnke, Gregory W. Brown, William Waller, 2008, How Important is Financial Risk, Working paper, Retrieved from Social Science Research Network.

[6] Brown, Gregory, Robert Harris, Wendy Hu, Tim Jenkinson, Steven Kaplan, David Robinson, 2020, Private equity portfolio companies: A first look at Burgiss holdings data, Working Paper, Retrieved from Social Science Research Network.

[7] Kaul, Nary and Singh, Who Does Private Equity Buy? Evidence on the role of private equity from buyouts of divested businesses, 2017, Strategic Management Journal

[8] Bansraj, Smit, Volosovych, 2019, Can Private Equity Funds act as Stategic Buyers?. Working Paper. Retrieved from Social Science Research Network.

[9] Boucly, Quentin, David Sraer, David Thesmar, 2011, Growth LBOs, Journal of Financial Economics, 102, 432-453

[10] Florian Szucs, 2013, MA and RD: Assymetric effects on Targets and Acquirers?, Working Paper. Retrieved from Social Science Research Network.

[11] Bharath and Dittmar, Why do Firms use Private Equity to Opt out of Public Markets?, 2010, Review of Financial Studies

[12] Jensen, 1989, Eclipse of the Public Corporation, Harvard Business Review Jenkinson and Sousa, What determines the exit decision of leveraged buyouts? 2015, Working Paper

[13] Jenkinson and Stucke, 2011, Who benefits from the leverage in LBOs? Working

paper

[14] Hammer, Knaer, Pfucke and Schwetzler, Inorganic growth strategies and evolution of the private equity business model, 2017, Journal of Corporate Finance

[15] Flannery, Mark J., Jan Hanousek, Anastasiya Shamshur, Jiri Tresl, 2020, MA Acitvity and the Capital Structure of Target Firms, Working Paper,ISSN 1211-3298

[16] Bradley, Michael, Gregg A. Jarrel, E. Han Kim, 1984, On the Existence of Optimal Capital Structure: Theory and Evidence, The Journal of Finance, 39, pp 957-878

[17] Goyal, Frank, Vidhan K. Goyal, 2007, Trade-off and Pecking Order Theories of Debt, Working Paper

[18] Rajan, Raghuram, Luigi Zingales, 1995, What Do We Know About Capital Structure? Some Evidence from International Data, The Journal of Finance, 50, 1421 - 1460

[19] Kaplan, Steve, Per Stromberg, 2009, Leveraged Buyout and Private Equity, Journal of Economic Perspectives, 23, 121-143

[20] Axelson, Ulf, Tim Jenkinson, Per Stromberg, Michael Weisbach, 2013, Borrow Cheap, Buy High? The Determinants of Leverage and Pricing in Buyouts, The Journal of Finance

[21] Cohn, Jonothan, Edith Hotchkiss, Erin Towery, 2020, The Motives for buyouts of private firms: Evidence from U.S. corporate tax returns, Working Paper

[22] Cohn, Jonothan, Edith Hotchkiss, Erin Towery, 2020, The Motives for buyouts of private firms: Evidence from U.S. corporate tax returns, Working Paper

[23] Cohn, Jonothan, Edith Hotchkiss, Erin Towery, 2020, The Motives for buyouts of private firms: Evidence from U.S. corporate tax returns, Working Paper

[24] Gupta, Atul, Sabrina T. Howell, Constantine Yannelis, Abhinav Gupta, 2020, Does Private Equity Investment in Healthcare Benefit Patients? Evidence from Nursing Homes, Working Paper

[25] Eaton, Charlie, Sabrina T. Howell, Constantine Yannelis, 2018, When Investor Incentives and Consumer Interests Diverge: Private Equity in Higher Education, The Review of Financial Studies, 33, 4024-4060

[26] Faulkender, Michael, Mark J. Flannery, Kristine Watson Hankins, Jason Smith, 2011, Journal of Financial Economics, 103, 632-646

[27] Imbens, Guido, Jeffrey Woolridge, 2007, Estimation of Average Treatment Effects

Under Unconfoundedness, NBER

[28] Mitton, Todd, 2006, Why Have Debt Ratios Increased For Firms in Emerging Markets?, Working paper, Retrieved from Social Science Research Network.

[29] Lerner, Josh, Morten Sorenson, Per Stromberg, 2011, Private Equity and Long-Run Investment: The Case of Innovation, The Journal of Finance, 66, pp. 445-477

[30] Leland, Hayne, 1994, Corporate Debt Value, Bond Covenants and Optimal Capital Structure. The Journal of Finance, 49, pp 1213-1252

[31] Leland, Hayne, Klaus Bjerre Toft, 1996, Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads. The Journal of Finance, 51, pp 987-1019

[32] Fracasi, Cesare, Allesandro Previtero, Albert Sheen, 2019, Barbarians at the Store, Working paper, Retrieved from Social Science Research Network.

[33] Merton, Robert C., Theory of Rational Option Pricing, 1973, The Bell Journal of Economics and Management Science, 4, 141 - 183

[34] Bernstein, Lerner and Mezanotti, 2017, Private Equity and Financial Fragility During the Crisis, NBER Working Paper, 23626

[35] De Maeseneire W., S. Brinkhuis, 2012, What drives leverage in leveraged buyouts? An analysis of European leveraged buyouts' capital structure, Accounting and Finance, 155-182

Appendix A: Does Private Equity Systematically Over-Lever Companies?

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Figure A7: **Event Study: PE Ownership and changes in Debt Structure**
Notes: This figure reports the dynamic difference-in-differences estimates of company-level outcomes around a PE buyout event as well as its confidence interval estimated in the full sample, using $t = -1$ as the baseline. The solid grey line separates post-buyout from pre-buyout period. Specifically, the $(\beta_s)_{s=-3,-2...3}$ of the following estimated equation are reported.

$$Y_{jt} = \alpha_j + \alpha_t + \sum_{s \neq -1} \beta_s (Deal\ Year_{js}) + X_{jt-1} + \epsilon_{jt}$$

$Deal\ Year_{js}$ is one in year s relative to the buyout year for firm j. X_{jt} is a vector of firm-level controls from Rajan-Zingales (1995). The outcome variables is labelled in the y-axis and defined as before. The control group is matched using propensity scores as outlined in section 4.2. Standard errors are clustered at the firm-level.

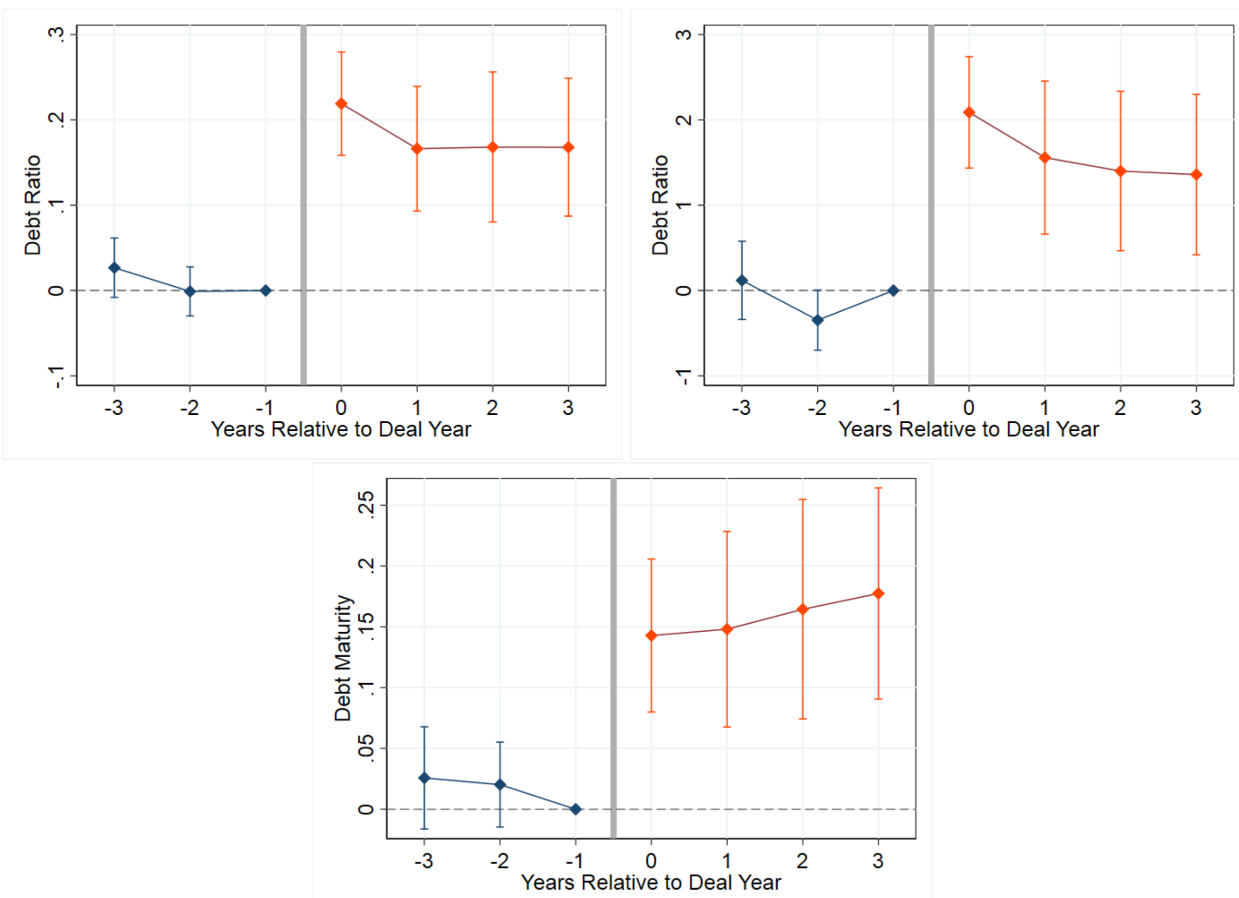

Table A1: **Logit Model of Private Equity Targeting**
$Notes$: This table shows estimates of the relationship between pre-deal firm characteristics and whether a firm is a target of a private equity leveraged buyout. The dependant variable is a dummy taking the value of 1 if a firm is acquired by a PE fund through a buyout, 0 otherwise. Pre-deal firm characteristics are obtained from the period $t = [-1, -3]$, where $t = 0$ refers to the year when a buyout event occurs. Consistent with previous studies explanatory variables include $log(Assets)$, $Sales\ Growth$ and $Asset\ Turnover$. I also include the $Interest\ Coverage$ ratio since it is likely to be correlated with the decision to acquire a firm. All specifications controls for year fixed effects. Standard errors are clustered at the firm-level.

	(1)	(2)	(3)	(4)
$log\ (Assets)$	-0.700***	-0.710***	-0.685***	-0.698***
	(0.069)	(0.074)	(0.070)	(0.076)
$Sales\ Growth$	-0.003***	-0.002***	-0.003***	-0.002***
	(0.001)	(0.001)	(0.001)	(0.001)
$Interest\ Coverage$		0.002***		0.002***
		(0.001)		(0.000)
$Asset\ Turnover$			0.026	0.021
			(0.020)	(0.019)
R^2	0.154	0.165	0.156	0.168
N	4879	4279	4824	4242

Standard errors in parentheses
$^{*}\ p < .10$, $^{**}\ p < .05$, $^{***}\ p < .01$

Table A2: **Propensity Score Matching Covariate Balance**
Notes: This table compares covariate balance before and after propensity score matching. Matching is based on log (Assets), Sales Growth, Asset Turnover and Interest Coverage within the same country-sector-year. Columns (1), (2), (5) and (6) report raw means of the outcomes for treated and propensity-score matched controls. Columns (3)/(6) and (4)/(8) presents p-values for the difference in means and medians using a T-test and a Wilcoxon Ranksum test respectively. The second panel reports the same statistics for other outcomes.

	Balance before Matching				Balance after Matching			
	PE	Non-PE	T-test	Ranksum	PE	Non-PE	T-test	Ranksum
Matching Covariates								
Log (Assets)	17.83	19.74	0.007***	0.000***	17.83	18.12	0.091*	0.120
Sales Growth	8.26	14.23	0.317	0.256	8.26	5.701	0.544	0.320
Asset Turnover	2.44	1.45	0.002***	0.000***	2.44	2.57	0.868	0.06*
Interest Coverage	75.72	25.26	0.000***	0.000***	75.72	52.82	0.246	0.121

* $p < .10$, ** $p < .05$, *** $p < .01$

A1: Leveraged Buyout Sample Representativeness

An important identification challenge is verifying that all deals retrieved are indeed leveraged buyouts undertaken by PE funds and not any other form of majority-owned private equity transaction. Ayash and Rastad (2018) survey the literature on LBOs, and suggest that researchers have difficulty differentiating between leveraged buyouts and other private equity investments. Since data providers typically cannot see deal leverage, they assume all buyouts are leveraged buyouts. They suggest it is possible a growth equity transaction can be majority-owned (i.e. a "growth equity buyout") and hence advocate using some form of a cut-off approach to filter out any non-LBO transactions, either based on transaction values or debt if capital structure is observable. Since I can observe firm-level debt, I use a cut-off approach based on debt itself, yielding a total of 814 unique LBOs [18]. Next, I go through my entire sample of deals and confirm that the acquirers are all PE funds. To ensure representativeness of the sample, I sort the top 200 deals on company assets during the deal year and randomly select 50 deals. These are reported in Tables A1 and A2 in the Appendix. As can be seen, these top deals are generally populated by well-known established PE funds such as KKR, Carlyle, Advent, Apollo, Blackstone, consistent with the conventional wisdom of the main players in the LBO market.

In addition to ensuring deal representatibility, I take the following steps. First, I follow the standard practice in the literature to retrieve and clean accounting data from Orbis in order to ensure national representativeness [19]. Second, I compare mean and median values of my variables with those from studies using other high-quality datasets and confirm that they are reasonably consistent. For example, researchers might be concerned that BvD data is relatively more focused on European firms and non-european data such as U.S. companies might suffer from some bias. However, once I verify the descriptive statistics for major non-European countries I find the data is consistent. For example, I

[18] I use various cut-off thresholds. I begin with a cut-off approach of debt ratio in the deal year is greater than debt ratio in the pre-deal year by at least some positive value such as 0.05, 0.08 and 0.1. My choice of cutoff does not affect the results.

[19] See instructions in Kalemli-Ozcan (2015)

compare U.S. company level data with papers using other datasets on the U.S. such as Cohn (2014) and Cohn (2020) and find that the data is quite comparable. Bartram, Brown and Waller (2013) use a sample of US firms with book value of Debt/Assets of 0.435 which is very similar to my sample. These values are described in more details in the next section covering descriptive statistics.

Table A3: **Deal Representativeness**

Notes: This Table (A1 and A2) reports 50 randomly selected deals from top 200 deals based on company assets. The list is sorted on deal size. The sample period is 2010-2019. Assets is company assets in the deal year. For deals with missing information on PE sponsor/Deal Value, I supplement information from Zephyr, Bloomberg, Pitchbook, LexisNexis and other public sources. Sponsoring PE funds in these top deals are mostly renowned private equity firms from US & Europe. Table A3 reports 25 randomly selected deals from bottom 200 deals based on company assets. Deals are sorted on assets since deal value was not available for these smaller deals.

PE Sponsor	Company Name	Country	Year	Deal Value ($ Mn)	Assets ($ Mn)
3G Capital	Kraft Foods Group	US	2015	28,000	23,000
The Blackstone Group	Thomson Reuters' Financial Data	CA	2017	17,000	26,400
KKR	Envision Healthcare Corp.	US	2018	9,900	17,000
Apollo	EP Energy Corp.	US	2012	6,720	8,300
Cerberus Capital	GE Money Bank SCA	FR	2017	4,600	2,100
Carlyle	Pharmaceutical Product Development	US	2011	3,480	2,000
Bain Capital	Skylark Co. Ltd.	JP	2011	3,400	2,900
Bain Capital	Kantar UK Ltd.	GB	2019	2,640	420
Montagu Private Equity	VISMA AS	NO	2017	2,400	2,400
CVC Capital Partners	TMF Orange Holding BV	NL	2018	2,400	1,700
Nordic Capital	Resurs Bank AB	SE	2012	2,200	1,300
Advantage Partners	Tokyo Star Bank Ltd.	JP	2011	2,200	28,000
Cinven Fith Fund	SYNLAB LABCO SA	FR	2015	2,160	1,100
CF Corporation	Fidelity & Gaurantee Life Inc.	US	2017	1,800	22,000
KDB Private Equity Fund	Daewoo Engineering	KR	2011	1,776	8,200
MBK Partners	ING Life Insurance Korea	KR	2013	1,600	23,000
Apollo	TAMINCO GROUP HOLDINGS SARL	LU	2012	1,560	400
KKR	LCY Chemical Corp.	TW	2019	1,560	1,600
The Blackstone Group	Luminor Bank	EE	2019	1,200	15,000
KKR	Serbia Broadband	RS	2014	1,200	550
GIP	Edinburgh Airport Ltd.	GB	2012	1,200	830
Cinven Sixth Fund	GENERALI LEBENSVERSICHERUNG AG	DE	2019	1,170	54,000
Ford Financial Fund	Mechanics Bank	US	2015	1,000	3,600
CDH Investment Advisory	FUJIAN NANPING NANFU BATTERY CO.	CN	2014	1,000	270
Permira	P&I PERSONAL & INFORMATIK AG	DE	2016	994	160

Table A4: Deal Representativeness Ctd.

PE Sponsor	Company Name	Country	Year	Deal Value ($ Mn)	Assets ($ Mn)
Torreal	SABA Infrastructures SA	ES	2011	970	620
Warburg Pincus	Endurance International Group	US	2011	900	1,200
Advent	ICE SPA	IT	2019	840	170
Bain	GRUPO NOTREDAME	BR	2014	750	390
Axcel	DANMARKS SKIBSKREDIT A/S	DK	2016	710	8,900
BC Partners	Sabre Insurance Co. Ltd.	GB	2013	670	550
Affinity Equity Partners	LOCK&LOCK CO., LTD	KR	2017	590	680
Tikehau Capital Partners	VOYAGE HEALTHCARE GROUP LTD	GB	2014	568	220
3i	ONEMED GROUP OY	FI	2011	550	170
LBO France	IKKS GROUP SAS	FR	2015	504	260
HELIOS INVESTORS III LP	CROWN AGENTS BANK LTD	GB	2016	500	1,100
Varde Partners	CREST NICHOLSON PLC	GB	2011	500	1,100
Yufeng Capital	ESAOTE SPA	IT	2018	480	420
Standard Charterd PLC	Union Bank of Nigeria Plc	NG	2012	456	6,700
Bridgepoint	CABB GMBH	DE	2011	439	97
Guggenheim	Equitrust Life Insurance	US	2011	430	7,200
Charterhouse Capital	DOC GENERICI SRL	IT	2013	410	520
KKR	Travelopia Holdings	GB	2017	400	420
Cinven	HEIDELBERGER AG	DE	2014	390	8,600
Pamlona Capital Mgt.	Partner in Food Hungaria	HU	2015	378	200
Baring Private Equity Asia	Hexaware Technologies Ltd.	IN	2013	360	300
Silver Lake	Cegid Group SA	FR	2017	360	440
Apax Partners	CABOVISAO	PT	2016	360	240
Ardian	SIACI SAINT HONORE SAS	FR	2015	320	290
BC Partners LTD	Nille AS	NO	2011	307	66
Nordic Capital	VIZRT LTD	IL	2015	300	350
LBO France	Chryso SAS	FR	2014	300	280

Table A5: Deal Representativeness Ctd. (smaller deals)

PE Sponsor	Company Name	Country	Year	Assets ($ Mn)
PAI Partners	ADB BVBA	BE	2013	177.0
PAI Partners	IPH - INDUSTRIAL PARTS HOLDING SAS	FR	2013	155.7
Carlyle Japan Partners	SANKYO RIKAGAKU CO., LTD	JP	2019	151.8
Bain Capital	MKM BUILDING SUPPLIES LTD	GB	2017	141.0
Morgan Stanley Private Equity	NOLBOO CO., LTD	KR	2011	99.8
3I Group Plc.	EURO-DIESEL SA	BE	2016	81.2
Phoenix Equity Partners	NEXUS VEHICLE MANAGEMENT LTD	GB	2018	62.2
Axa Investment Managers Private Equity Europe Sa.	BALTCOM TV SIA	LV	2011	54.7
Capman Buyout Fund	HARVIA OY	FI	2014	51.8
Alto Capital	MILLEFILI SPA	IT	2018	49.9
Ergon Capital Partners	GROEP DE BOECK	BE	2011	49.5
Apax Partners LLP	VOCALCOM SA	FR	2011	44.3
TA Associates	CMOSIS NV	BE	2014	36.6
TPG	VICTORIA PLUM LTD	GB	2014	35.0
Alcuin Capital Partners LLP	KRISPY KREME UK LTD	GB	2011	30.6
Montagu Private Equity	FSP ACQUISITION LTD	GB	2016	27.9
Darby Overseas Investment Ltd.	GRAMEX 2000 KERESKEDELMI KFT	HU	2014	22.6
Mml Capital Partners	LOWE REFRIGERATION LTD	GB	2014	22.1
Providence Equity Partners	ISTITUTO MARANGONI SRL	IT	2011	21.5
Carlyle Europe Technology Partners	ITRS GROUP LTD	GB	2011	18.6
Apax Partners	IDEALISTA LIBERTAD Y CONTROL SA	ES	2015	18.0
Gilde Buy Out Partners Bv.	OYSTERSHELL SA/NV	BE	2017	17.5
Adelis Equity Partners Fund I Ab.	MED GROUP OY	FI	2014	14.6
LBO France	SERAPID FRANCE SAS	FR	2017	14.4
Investindustrial Growth LP	VAIMO SRL	IT	2018	13.1
Equistone Partners Europe Ltd.	BFT MASTCLIMBING LTD	GB	2017	10.9